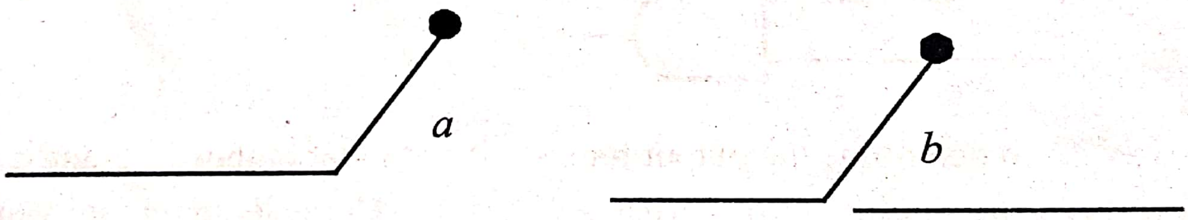
circuit as shown above.

2.5.3 Application of Boolean Algebra of Switching Circuits

The most important application of Boolean algebra is in the field of electrical circuit theory and particularly in switching circuit. The simplest switching device is the ordinary off-on or open-closed or two state designated by 0 and 1. We know that Switches can be combined in series or parallel connection and the behaviour of such combination depends on Boolean logic

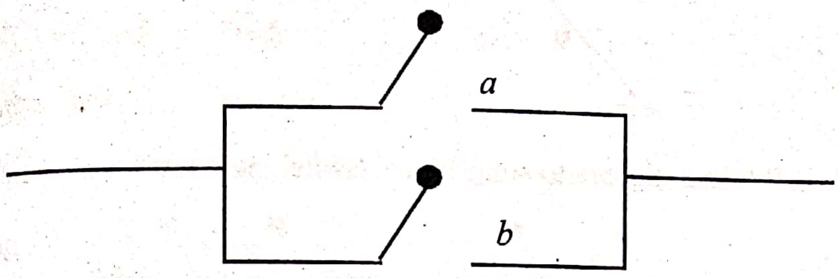
For example : A series connection is closed if and only if both are closed and a parallel connection is closed if and only if atleast one of them is closed

Remark . (1) Two Switches a and b are connected in series



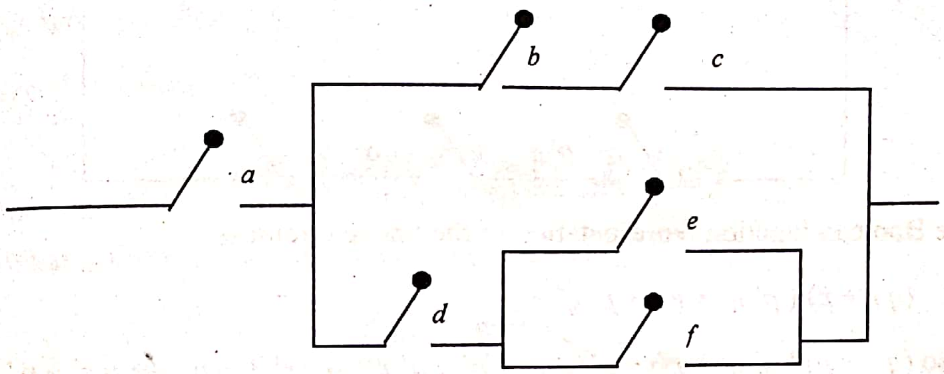
is represented by $a \wedge b$ or $a . b$

(2) Two switches a and b are connected in parallel



is represented by $a \vee b$ or $a + b$

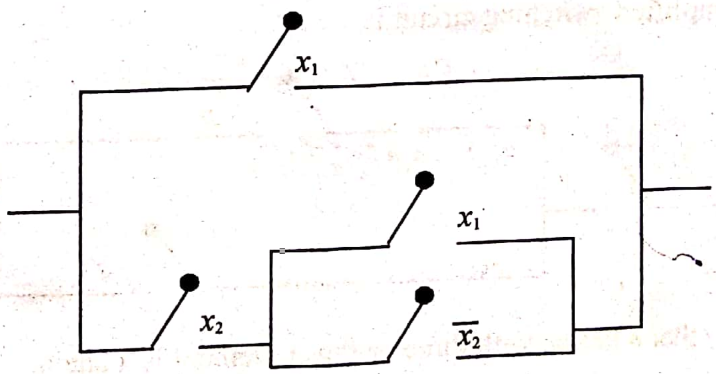
Example 3. Find the Boolean function to represent the following circuit



Sol. The Boolean function to represent the given circuit is

$$a\,[b\,c + d\,(e + f)] \text{ or } a \wedge [(b \wedge c) \vee (d \wedge (e \vee f))]$$

Example 4. Write the Boolean function represented by the following circuit



$x_2(x_1 + \bar{x}_2)$

Sol. The Boolean function representation by the given circuit is $x_1 + x_2\,(x_1 \cdot \bar{x}_2)$

Remark : The above Boolean function representation can be reduced to x_1 as

$$x_1 + x_2\,(x_1 + \bar{x}_2) = x_1 + (x_2\,x_1) + x_2\bar{x}_2$$

$$= x_1 + x_2 x_1 = 1\cdot x_1 + x_2 x_1$$

$$= (1 + x_2)\,x_1 = 1\cdot x_1 = x_1$$

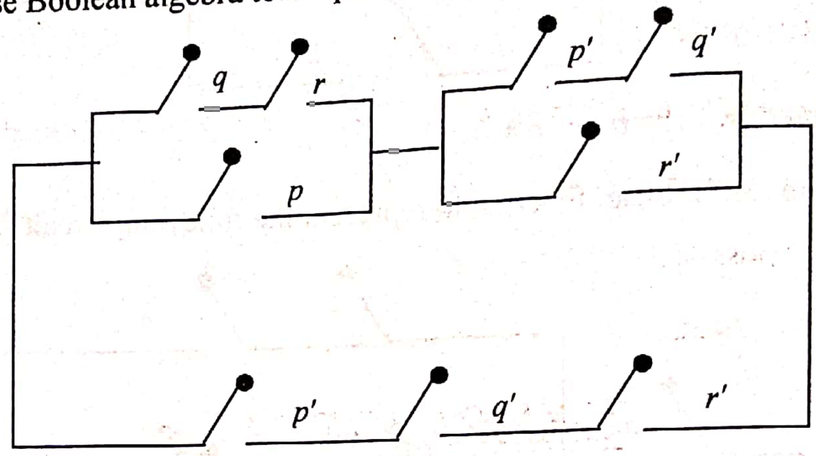
$\because 1 + a = a$
$\forall a$

$$= x_1 + 0\,x_1$$

$$= x_1 + 0$$

$$= x_1$$

Example 5. Use Boolean algebra to simplify the Switching circuit



Sol. The Boolean function representation of the above circuit is

$$(q\,r + p)\,(p'\,q' + r') + p'\,q'\,r'$$

Also $(q\,r + p)\,(p'\,q' + r') + p'\,q'\,r' = p'\,q\,q'\,r + q\,r\,r' + p\,p'\,q' + p\,r' + p'\,q'\,r'$

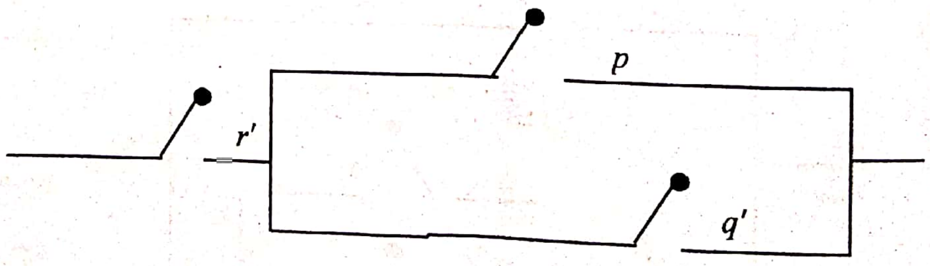
$$= p'\,(0)\,r + q\,(0) + (0)\,q' + p\,r' + p'\,q'\,r'$$

$$= (p + p'\,q')\,r'$$

$$= (p + p')\,(p + q')\,r'$$

$$= (p + q')\,r' \qquad p + p' = 1$$

The simplified switching circuit is



Example 6. Show that a lattice with three or fewer element is a chain.

Sol. Let (L, ≤) be a lattice. If n (L) = 1, then L is trivially a chain.

Let $n\,(L) = 2$ and $L = \{a, b\}$

$\therefore$ sup $\{a, b\}$ is either a or b

Let sup $\{a, b\} = a$

$\therefore \quad b \le a$

$\therefore$ $(L, \leq)$ is a chain.

Let $n(L) = 3$ and $L = \{a, b, c\}$.

If possible let L is not a chain.

$\therefore$ $\exists$ non-comparable element elements say a and b such that neither $a \leq b$ nor $b \leq a$

$\therefore$ $\sup\{a, b\} = c$ and $\text{Inf}\{a, b\} = c$

$\Rightarrow$ $a \leq c, b \leq c$ and $c \leq a, c \leq b$

$\Rightarrow$ $a = c, b = c \Rightarrow a = b = c.$

This is not possible.

Hence l is a chain.